Quantum Tunneling of Stellar Remnants: Calculating Decay Rates of Degenerate Astrophysical Objects

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Abstract & Background

When stars exhaust their nuclear fuel, they collapse into one of three objects: white dwarfs, neutron stars, or black holes. This project investigates what physical conditions allow each of these objects to exist, and what ultimately determines their mass limits and lifetimes.

During stellar evolution, the internal pressure and gravity are at equilibrium until the star exhausts its thermonuclear fusion supply. Smaller stars ending their main sequence stage below 8 solar masses stabilize as white dwarfs with masses up to 1.44 solar masses. More massive stars ending their main sequence at up to 25 solar masses collapse into neutron stars ranging from 1.3 to 3 solar masses. Stars exceeding 25 solar masses will collapse into black holes.

To better understand the transitional boundaries and conditions between the different final stages requires knowledge of both classical and relativistic physics to describe how matter behaves under extreme densities and gravitational fields.

Equations of State

White & Black Dwarfs

Relativistic Fermi Gas

$$P = \frac{8\pi}{3h^3}\int_0^{\hat{p}} \frac{p^4}{\sqrt{p^2c^2 + m_e^2c^4}}\,dp$$

Mass (Distant Observer)

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

Lane-Emden Equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n$$

Neutron Stars

Fermi Momentum Parameter

$$t = 4\sinh\left(\frac{\hat{p}}{m_n c}\right)$$

Density

$$\rho = Kc^2(\sinh t - t)$$

Pressure

$$P = \frac{1}{3}K\left(\sinh t - 8\sinh\frac{t}{2} + 3t\right) \qquad K = \frac{\pi m_n^4 c^5}{4h^3}$$

Tolman-Oppenheimer-Volkoff Equation

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

Black Holes

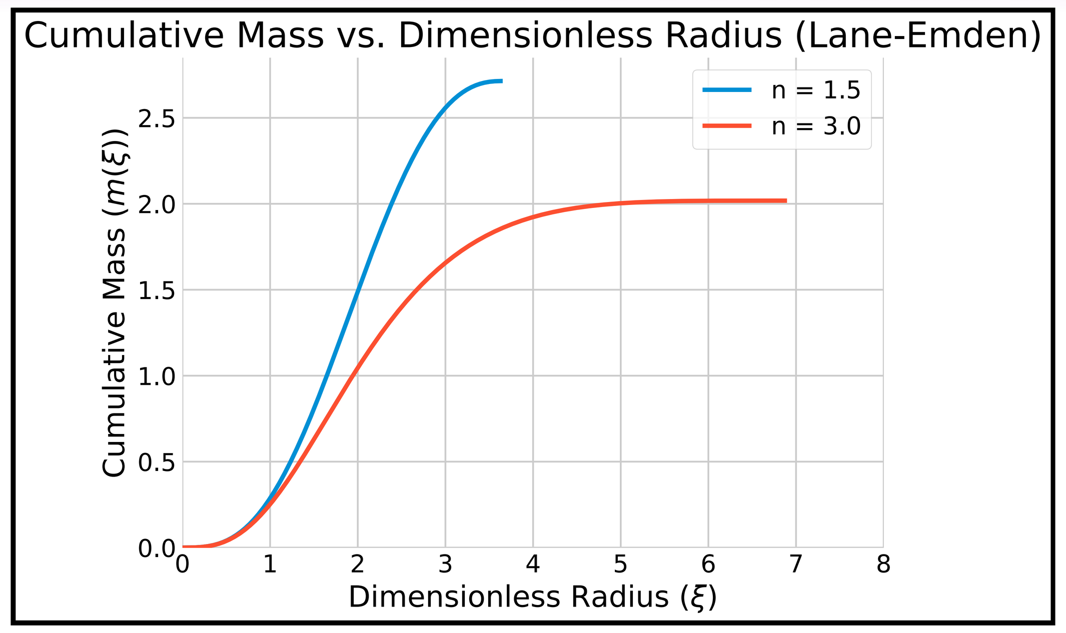
Schwarzschild Radius

$$R_s = \frac{2GM}{c^2}$$

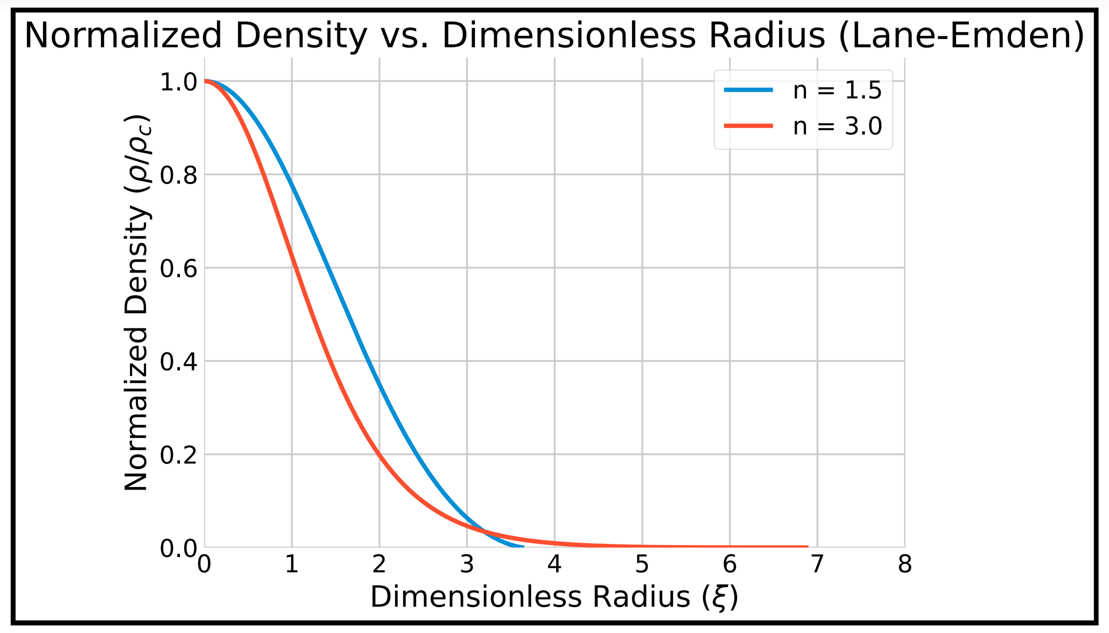
Hawking Radiation Time

$$\tau = \frac{5120\pi G^2 M^3}{\hbar c^4}$$

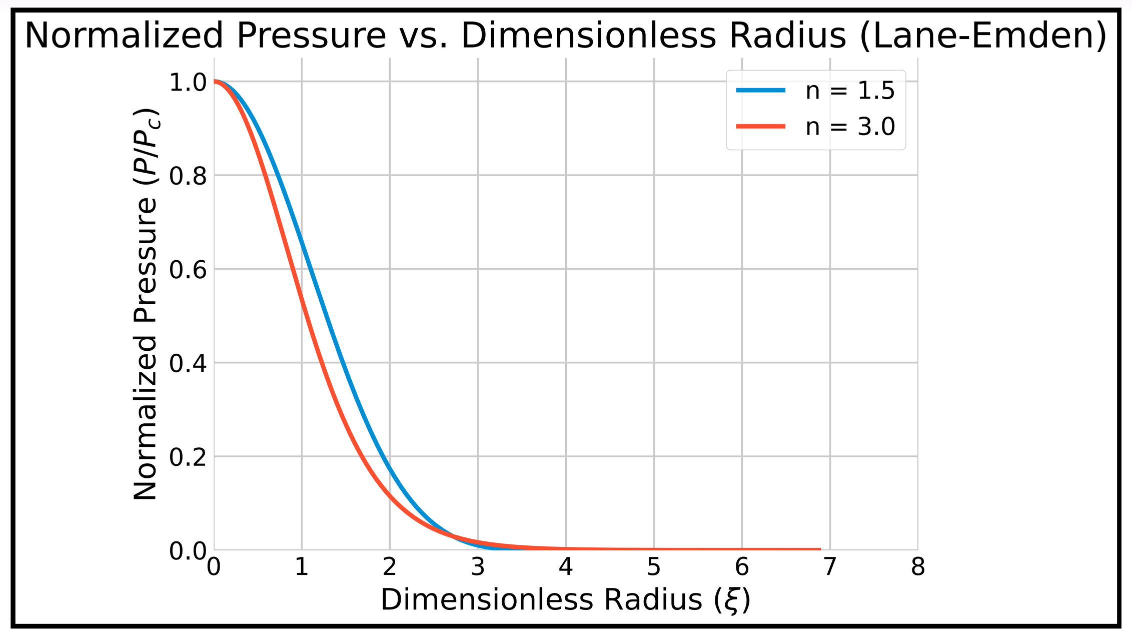
White & Black Dwarfs



The polytropic index n ($0 \le n \le 5$) describes how mass is distributed with radius inside a star. Smaller values of n correspond to a steeper initial rise, meaning mass accumulates more rapidly at smaller radii.

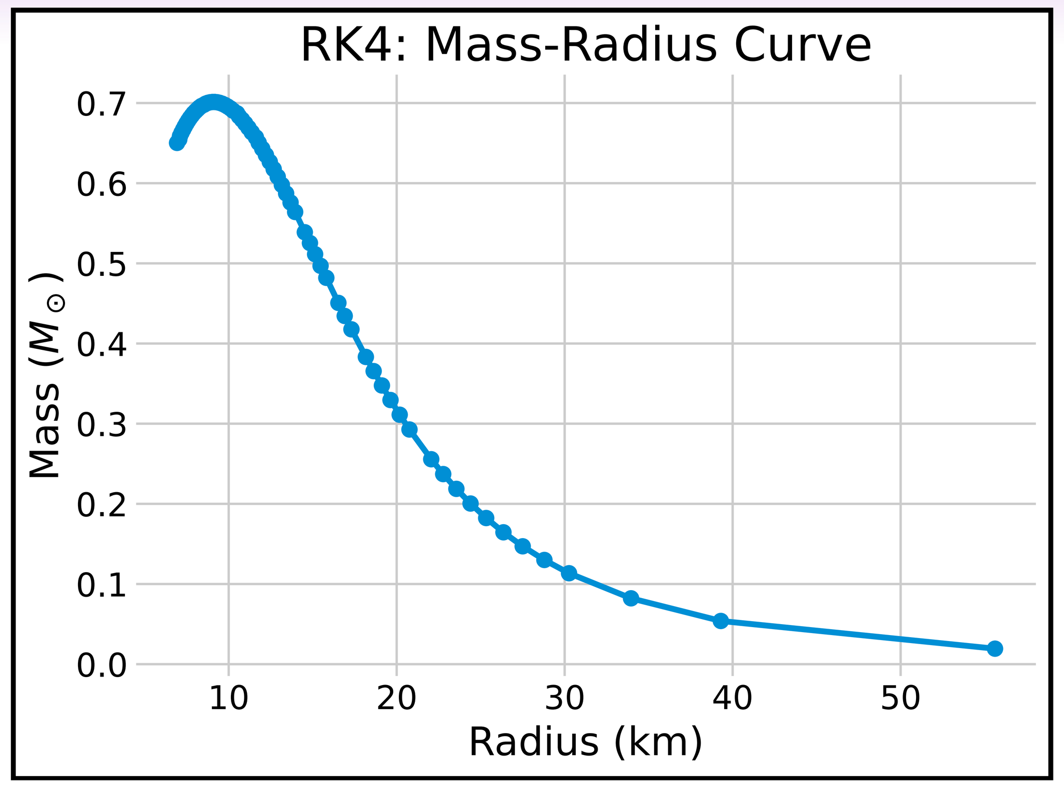


The normalized density, defined as the ratio of density at a given radius to the central density, describes how density is distributed throughout a star's interior. Smaller values of n correspond to a more uniform density distribution, where density changes gradually from the center to the surface.

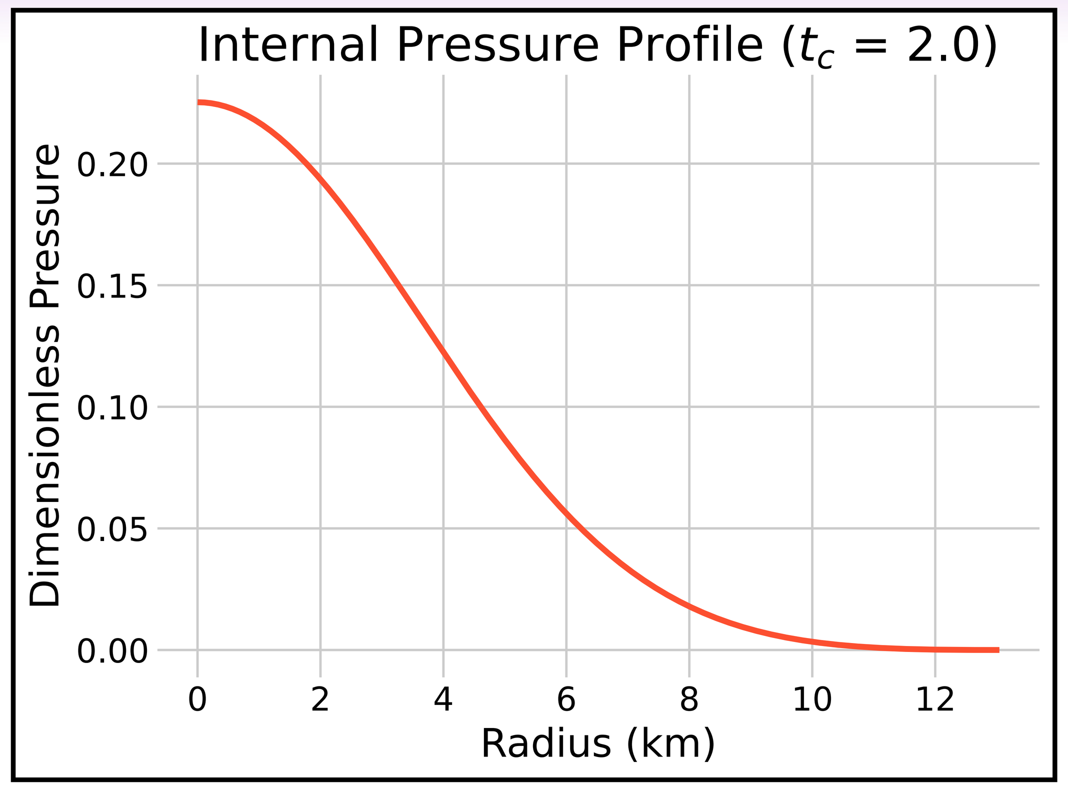


The normalized pressure, defined as the ratio of pressure at a given radius to the central pressure, describes how pressure is distributed throughout a star's interior. Pressure decreases as the radius increases. Higher values of n show a steeper pressure drop, which reflects a less uniform pressure distribution.

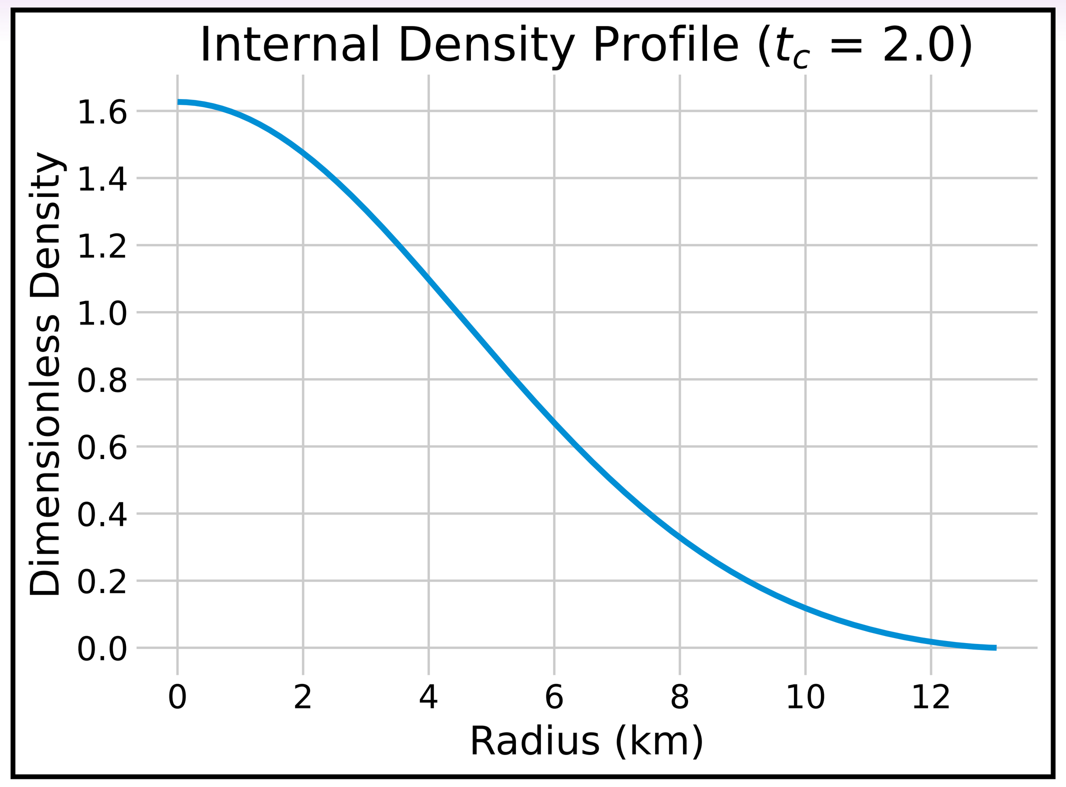
Neutron Stars



The mass-radius curve describes the relationship between a neutron star's total mass and its radius. Mass increases with radius up to a peak and then asymptotically decreases, which reflects how more compressed stars can support greater masses up to a certain limit.



The internal pressure profile, defined as the dimensionless pressure at a given radius, describes how pressure is distributed throughout a neutron star. Pressure decreases as radius increases to maintain hydrostatic equilibrium.



The internal density profile, defined as the dimensionless density at a given radius, describes how density is distributed throughout a neutron star. Density decreases with increasing radius, equaling zero at the boundary of the star.

Methods

To study white dwarfs, we employed the Chandrasekhar limit equation using appropriate Lane-Emden index values for hydrostatic equilibrium, which describes the balance between gravity and electron degeneracy pressure.

For neutron stars, their density is far too great to be calculated using Newtonian mechanics, so we have to employ the general relativistic extension of the same idea; the Tolman-Oppenheimer-Volkoff (TOV) equation for relativistic hydrostatic equilibrium.

In the cases of white dwarfs and neutron stars, we used the fourth-order Runge-Kutta (RK4) numerical method to solve the resulting systems of differential equations and generate structural profiles of density, pressure, and stellar radius.

For black holes, we examined the Schwarzschild radius, which defines the event horizon of a non-rotating black hole, and explored Hawking radiation as a mechanism for their eventual decay.

References

[1] Selçuk Bayin. "Solutions of Einstein's field equations for static fluid spheres". In: *Phys. Rev. D* 18 (Oct. 1978). doi: 10.1103/PhysRevD.18.2745.

[2] M E Caplan. "Black dwarf supernova in the far future". In: *Monthly Notices of the Royal Astronomical Society* 497.4 (Aug. 2020), pp. 4357–4362. issn: 1365-2966. doi: 10.1093/mnras/staa2262. url: http://dx.doi.org/10.1093/mnras/staa2262.

[3] S. Chandrasekhar and E. A. Milne. "The Highly Collapsed Configurations of a Stellar Mass". In: *Monthly Notices of the Royal Astronomical Society* 91.5 (Mar. 1931), pp. 456–466. issn: 0035-8711. doi: 10.1093/mnras/91.5.456. eprint: https://academic.oup.com/mnras/article-pdf/91/5/456/3079698/mnras91-0456.pdf. url: https://doi.org/10.1093/mnras/91.5.456.

[4] Jeffrey D. Cummings et al. "The White Dwarf Initial–Final Mass Relation for Progenitor Stars from 0.85 to 7.5 M". In: *The Astrophysical Journal* 866.1 (Oct. 2018), p. 21. doi: 10.3847/1538-4357/aadfd6. url: https://doi.org/10.3847/1538-4357/aadfd6.

[5] Jasonjot S. Kalirai et al. "The Initial-Final Mass Relation: Direct Constraints at the Low-Mass End". In: *The Astrophysical Journal* 676.1 (Mar. 2008). doi: 10.1086/527028. url: https://doi.org/10.1086/527028.

[6] J. R. Oppenheimer and G. M. Volkoff. "On Massive Neutron Cores". In: *Phys. Rev.* 55 (4 1939), pp. 374–381. doi: 10.1103/PhysRev.55.374. url: https://link.aps.org/doi/10.1103/PhysRev.55.374.

[7] David G. Robertson. "The Structure of White Dwarf and Neutron Stars: Instructor Notes". In: *Compendium of Graduate-Level Physics Problems*. Otterbein University / Physics Education Research, 2009. url: https://api.semanticscholar.org/CorpusID:131766133.

[8] Richard C. Tolman. "Static Solutions of Einstein's Field Equations for Spheres of Fluid". In: *Phys. Rev.* 55 (4 1939), pp. 364–373. doi: 10.1103/PhysRev.55.364. url: https://link.aps.org/doi/10.1103/PhysRev.55.364.