Table of Integrals

A constant of integration is understood to be added to all indefinite integrals.

Rational Functions

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} \qquad (n\neq -1)$$
$$\int x(ax+b)^n\,dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} \qquad (n\notin\{-1,-2\})$$
$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$$
$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$
$$\int \frac{x^2}{ax+b}\,dx = \frac{1}{a^3}\left(\frac12(ax+b)^2 - 2b(ax+b) + b^2\ln|ax+b|\right)$$
$$\int \frac{dx}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$
$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$
$$\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$
$$\int \frac{x^2}{(ax+b)^2}\,dx = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$
$$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left|\frac{x}{ax+b}\right|$$
$$\int \frac{dx}{x^2(ax+b)^2} = -\frac{a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left|\frac{ax+b}{x}\right|$$
$$\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\ln\left|\frac{px+q}{ax+b}\right|$$
$$\int \frac{x}{(ax+b)(px+q)}\,dx = \frac{1}{bp-aq}\left(\frac{b}{a}\ln|ax+b| - \frac{q}{p}\ln|px+q|\right)$$
$$\int \frac{dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left(\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left|\frac{px+q}{ax+b}\right|\right)$$
$$\int \frac{x}{(ax+b)^2(px+q)}\,dx = \frac{1}{bp-aq}\left(\frac{q}{bp-aq}\ln\left|\frac{ax+b}{px+q}\right| - \frac{b}{a(ax+b)}\right)$$
$$\int \frac{ax+b}{px+q}\,dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln|px+q|$$
$$\int \frac{dx}{x^2\pm a^2} = \pm\frac{1}{a}\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a} = \frac12\ln\left|\frac{x+a}{x-a}\right|\end{cases} \qquad (|x|>|a|)$$
$$\int \frac{x}{x^2\pm a^2}\,dx = \frac12\ln|x^2\pm a^2|$$
$$\int \frac{x^2}{x^2\pm a^2}\,dx = x\mp a\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a}\end{cases}$$
$$\int \frac{dx}{x(x^2\pm a^2)} = \pm\frac{1}{2a^2}\ln\left(\frac{x^2}{x^2\pm a^2}\right)$$
$$\int \frac{dx}{x^2(x^2\pm a^2)} = \mp\frac{1}{a^2x}\mp\frac{1}{a^3}\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a}\end{cases}$$
$$\int \frac{dx}{(x^2\pm a^2)^2} = \pm\frac{x}{2a^2(x^2\pm a^2)} \pm \frac{1}{2a^3}\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a}\end{cases}$$
$$\int \frac{x}{(x^2\pm a^2)^2}\,dx = -\frac{1}{2(x^2\pm a^2)}$$
$$\int \frac{x^2}{(x^2\pm a^2)^2}\,dx = -\frac{x}{2(x^2\pm a^2)} + \frac{1}{2a}\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a}\end{cases}$$
$$\int \frac{dx}{x(x^2\pm a^2)^2} = \pm\frac{1}{2a^2(x^2\pm a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2\pm a^2}\right)$$
$$\int \frac{dx}{x^2(x^2\pm a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2\pm a^2)} - \frac{3}{2a^5}\begin{cases}\tan^{-1}\frac{x}{a}\\ \coth^{-1}\frac{x}{a}\end{cases}$$
$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| = \frac{1}{a}\tanh^{-1}\frac{x}{a} \qquad (|x|<|a|)$$
$$\int \frac{x}{a^2-x^2}\,dx = -\frac12\ln(a^2-x^2)$$
$$\int \frac{x^2}{a^2-x^2}\,dx = -x + a\tanh^{-1}\frac{x}{a}$$
$$\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$$
$$\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{a^3}\tanh^{-1}\frac{x}{a}$$
$$\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{2a^3}\tanh^{-1}\frac{x}{a}$$
$$\int \frac{x}{(a^2-x^2)^2}\,dx = \frac{1}{2(a^2-x^2)}$$
$$\int \frac{x^2}{(a^2-x^2)^2}\,dx = \frac{x}{2(a^2-x^2)} - \frac{1}{2a}\tanh^{-1}\frac{x}{a}$$
$$\int \frac{dx}{x(a^2-x^2)^2} = \frac{1}{2a^2(a^2-x^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$$
$$\int \frac{dx}{x^2(a^2-x^2)^2} = -\frac{1}{a^4x} + \frac{x}{2a^4(a^2-x^2)} + \frac{3}{2a^5}\tanh^{-1}\frac{x}{a}$$
$$\int \frac{dx}{ax^2+bx+c} = \begin{cases}\frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} & (4ac-b^2>0)\\ -\frac{2}{\sqrt{b^2-4ac}}\tanh^{-1}\frac{2ax+b}{\sqrt{b^2-4ac}} = \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| & (4ac-b^2<0)\\ -\frac{2}{2ax+b} & (4ac-b^2=0)\end{cases}$$
$$\int \frac{x}{ax^2+bx+c}\,dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$
$$\int \frac{x^2}{ax^2+bx+c}\,dx = \frac{x}{a} - \frac{b}{2a^2}\ln|ax^2+bx+c| + \frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}$$
$$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int\frac{dx}{ax^2+bx+c}$$
$$\int \frac{dx}{x^2(ax^2+bx+c)} = \frac{b}{2c^2}\ln\left|\frac{ax^2+bx+c}{x^2}\right| - \frac{1}{cx} + \frac{b^2-2ax}{2c^2}\int\frac{dx}{ax^2+bx+c}$$

Involving $\sqrt{x^2 \pm a^2}$

$$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \begin{cases}\sinh^{-1}\frac{x}{a} = \ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|\\ \cosh^{-1}\left|\frac{x}{a}\right| = \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|\end{cases}$$
$$\int \frac{x}{\sqrt{x^2\pm a^2}}\,dx = \sqrt{x^2\pm a^2}$$
$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}\,dx = \frac12 x\sqrt{x^2\pm a^2} \mp \frac{a^2}{2}\begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{dx}{x\sqrt{x^2\pm a^2}} = \mp\frac{1}{a}\begin{cases}\operatorname{csch}^{-1}\frac{x}{a}\\ \sec^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{dx}{x^2\sqrt{x^2\pm a^2}} = \mp\frac{\sqrt{x^2\pm a^2}}{a^2x}$$
$$\int \sqrt{x^2\pm a^2}\,dx = \frac12 x\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int x\sqrt{x^2\pm a^2}\,dx = \frac13\left(x^2\pm a^2\right)^{3/2}$$
$$\int x^2\sqrt{x^2\pm a^2}\,dx = \frac14 x\left(x^2\pm a^2\right)^{3/2} \mp \frac{a^2}{8}x\sqrt{x^2\pm a^2} - \frac{a^4}{8}\begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{\sqrt{x^2\pm a^2}}{x}\,dx = \sqrt{x^2\pm a^2} - a\begin{cases}\operatorname{csch}^{-1}\frac{x}{a}\\ \sec^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{\sqrt{x^2\pm a^2}}{x^2}\,dx = -\frac{\sqrt{x^2\pm a^2}}{x} + \begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{dx}{(x^2\pm a^2)^{3/2}} = \pm\frac{x}{a^2\sqrt{x^2\pm a^2}}$$
$$\int \frac{x}{(x^2\pm a^2)^{3/2}}\,dx = -\frac{1}{\sqrt{x^2\pm a^2}}$$
$$\int \frac{x^2}{(x^2\pm a^2)^{3/2}}\,dx = -\frac{x}{\sqrt{x^2\pm a^2}} + \begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \left(x^2\pm a^2\right)^{3/2}\,dx = \frac14 x\left(x^2\pm a^2\right)^{3/2} \pm \frac38 a^2x\sqrt{x^2\pm a^2} + \frac38 a^4\begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int x\left(x^2\pm a^2\right)^{3/2}\,dx = \frac15\left(x^2\pm a^2\right)^{5/2}$$
$$\int x^2\left(x^2\pm a^2\right)^{3/2}\,dx = \frac16 x\left(x^2\pm a^2\right)^{5/2} \mp \frac{a^2}{24}x\left(x^2\pm a^2\right)^{3/2} - \frac{a^4}{16}x\sqrt{x^2\pm a^2} \mp \frac{a^6}{16}\begin{cases}\sinh^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{(x^2\pm a^2)^{3/2}}{x}\,dx = \frac13\left(x^2\pm a^2\right)^{3/2} \pm a^2\sqrt{x^2\pm a^2} \mp a^3\begin{cases}\operatorname{csch}^{-1}\frac{x}{a}\\ \sec^{-1}\left|\frac{x}{a}\right|\end{cases}$$
$$\int \frac{(x^2\pm a^2)^{3/2}}{x^2}\,dx = -\frac{(x^2\pm a^2)^{3/2}}{x} + \frac32 x\sqrt{x^2\pm a^2} \pm \frac32 a^2\begin{cases}\operatorname{csch}^{-1}\frac{x}{a}\\ \cosh^{-1}\left|\frac{x}{a}\right|\end{cases}$$

Riemann Sum: $\int_a^b f(x)\,dx = \lim_{\max\Delta x_k\to0}\sum_{k=1}^n f(x_k^*)\Delta x_k$, where $x\in[a,b]$, and x_k^* is an arbitrary point in the interval $\Delta x_k = x_k - x_{k-1}$, $\{x_k\}$ being a partition of the interval $[a,b]$.
First Fundamental Theorem of Calculus: Let f be a continuous real-valued function defined on a closed interval $[a,b]$. Let F be the function defined, for all $x\in[a,b]$, by $F(x) = \int_a^x f(t)\,dt$. Then, F is continuous on $[a,b]$, differentiable on (a,b), and $F'(x) = f(x)$ for all $x\in(a,b)$.
Second Fundamental Theorem of Calculus: Let f be a real-valued function defined on a closed interval $[a,b]$ and F a real-valued function such that for all $x\in[a,b]$, $f(x) = F'(x)$. If f is integrable on $[a,b]$, then $\int_a^b f(x)\,dx = F(b)-F(a)$.
Mean Value Theorem: If $g:[a,b]\to\mathbb{R}$ is a continuous function and $f:[a,b]\to[0,\infty)$ is an integrable function, then there exists a number $c\in[a,b]$ such that $\int_a^b g(t)f(t)\,dt = g(c)\int_a^b f(t)\,dt$. In particular, if $f(t)=1$ for all $t\in[a,b]$, then there exists $c\in[a,b]$ such that $\int_a^b g(t)\,dt = g(c)(b-a)$.

Involving $\sqrt{a^2 - x^2}$ $\quad(|x| < |a|)$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$
$$\int \frac{x}{\sqrt{a^2-x^2}}\,dx = -\sqrt{a^2-x^2}$$
$$\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = -\frac12 x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$
$$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac1a\operatorname{sech}^{-1}\frac{x}{a} = -\frac1a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$
$$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$$
$$\int \sqrt{a^2-x^2}\,dx = \frac12 x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$
$$\int x\sqrt{a^2-x^2}\,dx = -\frac13\left(a^2-x^2\right)^{3/2}$$
$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac14 x(a^2-x^2)^{3/2} + \frac{a^2}{8}x\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a}$$
$$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\operatorname{sech}^{-1}\frac{x}{a}$$
$$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \sin^{-1}\frac{x}{a}$$
$$\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$
$$\int \frac{x}{(a^2-x^2)^{3/2}}\,dx = \frac{1}{\sqrt{a^2-x^2}}$$
$$\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{a^2-x^2}} - \sin^{-1}\frac{x}{a}$$
$$\int \frac{dx}{x(a^2-x^2)^{3/2}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3}\operatorname{sech}^{-1}\frac{x}{a}$$
$$\int \frac{dx}{x^2(a^2-x^2)^{3/2}} = -\frac{\sqrt{a^2-x^2}}{a^4x} + \frac{x}{a^4\sqrt{a^2-x^2}}$$
$$\int (a^2-x^2)^{3/2}\,dx = \frac14 x(a^2-x^2)^{3/2} + \frac{3a^2}{8}x\sqrt{a^2-x^2} + \frac38 a^4\sin^{-1}\frac{x}{a}$$
$$\int x(a^2-x^2)^{3/2}\,dx = -\frac15(a^2-x^2)^{3/2}$$
$$\int x^2(a^2-x^2)^{3/2}\,dx = -\frac16 x(a^2-x^2)^{5/2} + \frac{a^2}{24}x(a^2-x^2)^{3/2} + \frac{a^4}{16}x\sqrt{a^2-x^2} + \frac{a^6}{16}\sin^{-1}\frac{x}{a}$$
$$\int \frac{(a^2-x^2)^{3/2}}{x}\,dx = \frac13(a^2-x^2)^{3/2} + a^2\sqrt{a^2-x^2} - a^3\operatorname{sech}^{-1}\frac{x}{a}$$
$$\int \frac{(a^2-x^2)^{3/2}}{x^2}\,dx = -\frac{(a^2-x^2)^{3/2}}{x} - \frac32 x\sqrt{a^2-x^2} - \frac32 a^2\sin^{-1}\frac{x}{a}$$

Involving $\sqrt{ax + b}$

$$\int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a}\sqrt{ax+b}$$
$$\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2}{3a^2}(ax-2b)\sqrt{ax+b}$$
$$\int \frac{x^2}{\sqrt{ax+b}}\,dx = \frac{2}{15a^3}(3a^2x^2-4abx+8b^2)\sqrt{ax+b}$$
$$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases}-\frac{2}{\sqrt b}\coth^{-1}\sqrt{\frac{ax+b}{b}} & (b>0,\ ax>0)\\ -\frac{2}{\sqrt b}\tanh^{-1}\sqrt{\frac{ax+b}{b}} & (b>0,\ ax<0)\\ \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}} & (b<0)\end{cases}$$
$$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int\frac{dx}{x\sqrt{ax+b}}$$
$$\int \sqrt{ax+b}\,dx = \frac{2}{3a}(ax+b)^{3/2}$$
$$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(3ax-2b)(ax+b)^{3/2}$$
$$\int x^2\sqrt{ax+b}\,dx = \frac{2}{105a^3}(15a^2x^2-12abx+8b^2)(ax+b)^{3/2}$$
$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int\frac{dx}{x\sqrt{ax+b}}$$
$$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int\frac{dx}{x\sqrt{ax+b}}$$
$$\int \frac{px+q}{\sqrt{ax+b}}\,dx = \frac{2}{3a^2}(apx+3aq-2bp)\sqrt{ax+b}$$
$$\int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases}-\frac{2}{\sqrt p\sqrt{bp-aq}}\coth^{-1}\sqrt{\frac{p(ax+b)}{bp-aq}} & (bp-aq>0,\ ax>0)\\ -\frac{2}{\sqrt p\sqrt{bp-aq}}\tanh^{-1}\sqrt{\frac{p(ax+b)}{bp-aq}} & (bp-aq>0,\ ax<0)\\ \frac{2}{\sqrt p\sqrt{aq-bp}}\tan^{-1}\sqrt{\frac{p(ax+b)}{aq-bp}} & (bp-aq<0)\end{cases}$$
$$\int \frac{\sqrt{ax+b}}{px+q}\,dx = \frac{2}{p}\sqrt{ax+b} - \frac{2}{p\sqrt p}\begin{cases}\sqrt{bp-aq}\coth^{-1}\sqrt{\frac{p(ax+b)}{bp-aq}} & (bp-aq>0,\ ax>0)\\ \sqrt{bp-aq}\tanh^{-1}\sqrt{\frac{p(ax+b)}{bp-aq}} & (bp-aq>0,\ ax<0)\\ \sqrt{aq-bp}\tan^{-1}\sqrt{\frac{p(ax+b)}{aq-bp}} & (bp-aq<0)\end{cases}$$
$$\int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases}\frac{2}{\sqrt{ap}}\ln\left(\sqrt{a(px+q)}+\sqrt{p(ax+b)}\right) & (ap>0)\\ \frac{2}{\sqrt{-ap}}\tan^{-1}\sqrt{\frac{-p(ax+b)}{a(px+q)}} & (ap<0)\end{cases}$$
$$\int \frac{x}{\sqrt{(ax+b)(px+q)}}\,dx = \frac{1}{ap}\sqrt{(ax+b)(px+q)} - \frac{bp+aq}{2ap}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$$
$$\int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$$
$$\int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{1}{a}\sqrt{(ax+b)(px+q)} + \frac{aq-bp}{2a}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$$
$$\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

Involving $\sqrt{ax^2 + bx + c}$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases}\frac{1}{\sqrt a}\ln\left|2\sqrt a\sqrt{ax^2+bx+c}+2ax+b\right| & (a>0)\\ \frac{1}{\sqrt a}\sinh^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} & (a>0,\ 4ac-b^2>0)\\ \frac{1}{\sqrt a}\ln|2ax+b| & (a>0,\ 4ac-b^2=0)\\ -\frac{1}{\sqrt{-a}}\sin^{-1}\frac{2ax+b}{\sqrt{b^2-4ac}} & (a<0,\ 4ac-b^2<0)\end{cases}$$
$$\int \frac{x}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$
$$\int \frac{x^2}{\sqrt{ax^2+bx+c}}\,dx = -\frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$
$$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases}-\frac{1}{\sqrt c}\ln\left(\frac{2\sqrt c\sqrt{ax^2+bx+c}+bx+2c}{x}\right) & (c>0)\\ -\frac{1}{\sqrt c}\sinh^{-1}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) & (c<0,\ 4ac-b^2<0)\\ -\frac{1}{\sqrt c}\ln\left|\frac{bx+2c}{x}\right| & (c>0,\ 4ac-b^2=0)\\ \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) & (c>0,\ 4ac-b^2>0)\end{cases}$$
$$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$
$$\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$
$$\int x\sqrt{ax^2+bx+c}\,dx = \frac{1}{3a}(ax^2+bx+c)^{3/2} - \frac{b(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$
$$\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{6ax-5b}{24a^2}(ax^2+bx+c)^{3/2} + \frac{5b^2-4ac}{16a^2}\int\sqrt{ax^2+bx+c}\,dx$$
$$\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int\frac{dx}{\sqrt{ax^2+bx+c}} + c\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$
$$\int \frac{\sqrt{ax^2+bx+c}}{x^2}\,dx = -\frac{1}{x}\sqrt{ax^2+bx+c} + a\int\frac{dx}{ax^2+bx+c} + \frac{b}{2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

u-Substitution: $\int_a^b f(g(t))g'(t)\,dt = \int_{g(a)}^{g(b)} f(u)\,du$, where $u=g(t)$ and $du = g'(t)\,dt$.
Integration by Parts: $\int_a^b f(x)\,dg(x) = f(x)g(x)\big|_a^b - \int_a^b g(x)\,df(x)$.
Logarithm Integration: $\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$
Trigonometric Substitution: If integrand contains a^2-x^2, let $x=a\sin\theta$. If it contains $\sqrt{a^2+x^2}$, let $x=a\tan\theta$. If it contains $\sqrt{x^2-a^2}$, let $x=a\sec\theta$.
Partial Fractions: Degree of numerator must be larger than that of denominator. Factor denominator into first degree and irreducible quadratic factors. Set this equal to a sum of rational functions with each factor in the denominator, and a polynomial of degree one less with unknown coefficients in the numerator. Clear the fractions, equate coefficients of each power, and solve the system of equations for the coefficients. Example: $\frac{x}{(x+1)(x+2)^2(x^2+1)(x^2+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{x^2+2} + \frac{Hx+I}{(x^2+2)^2}$.
Derivative of Integral: If $F(x) = \int_{q(x)}^{p(x)} f(x,t)\,dt$, then $F'(x) = f(x,p(x))p'(x) - f(x,q(x))q'(x) + \int_{q(x)}^{p(x)}\frac{\partial}{\partial x}f(x,t)\,dt$.

Trigonometric Functions

$$\int \sin ax\,dx = -\frac1a\cos ax$$
$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$
$$\int x^2\sin ax\,dx = \frac{2x}{a^2}\sin ax + \left(\frac{2}{a^3}-\frac{x^2}{a}\right)\cos ax$$
$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax$$
$$\int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax$$
$$\int x^2\sin^2 ax\,dx = \frac{x^3}{6} - \left(\frac{x^2}{4a}-\frac{1}{8a^3}\right)\sin 2ax - \frac{x}{4a^2}\cos 2ax$$
$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad (|a|\neq|b|)$$
$$\int \frac{dx}{1\pm\sin ax} = \frac{2}{a\cot\frac{ax}{2}\pm a}$$
$$\int \frac{dx}{p+q\sin ax} = \begin{cases}\frac{2}{a\sqrt{p^2-q^2}}\tan^{-1}\frac{p\tan\frac{ax}{2}+q}{\sqrt{p^2-q^2}} & (|p|>|q|)\\ -\frac{2}{a\sqrt{q^2-p^2}}\coth^{-1}\frac{p\tan\frac{ax}{2}+q}{\sqrt{q^2-p^2}} & (|p|<|q|)\end{cases}$$
$$\int \frac{dx}{p^2+q^2\sin^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}}\tan^{-1}\left(\frac{\sqrt{p^2+q^2}}{p}\tan ax\right)$$
$$\int \frac{x}{1+\sin ax}\,dx = \frac{x}{a}\tan\left(\frac{ax}{2}-\frac{\pi}{4}\right) + \frac{2}{a^2}\ln\left|\cos\left(\frac{ax}{2}-\frac{\pi}{4}\right)\right|$$
$$\int \frac{x}{1-\sin ax}\,dx = \frac{x}{a}\cot\left(\frac{\pi}{4}-\frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\sin\left(\frac{\pi}{4}-\frac{ax}{2}\right)\right|$$
$$\int \cos ax\,dx = \frac1a\sin ax$$
$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$
$$\int x^2\cos ax\,dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a}-\frac{2}{a^3}\right)\sin ax$$
$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax$$
$$\int x\cos^2 ax\,dx = \frac{x^2}{4} + \frac{x}{4a}\sin 2ax + \frac{1}{8a^2}\cos 2ax$$
$$\int x^2\cos^2 ax\,dx = \frac{x^3}{6} + \left(\frac{x^2}{4a}-\frac{1}{8a^3}\right)\sin 2ax + \frac{x}{4a^2}\cos 2ax$$
$$\int \frac{dx}{1\pm\cos ax} = \pm\frac1a\begin{cases}\tan\frac{ax}{2}\\ \cot\frac{ax}{2}\end{cases}$$
$$\int \frac{dx}{p+q\cos ax} = \begin{cases}\frac{2}{a\sqrt{p^2-q^2}}\tan^{-1}\left(\sqrt{\frac{p-q}{p+q}}\tan\frac{ax}{2}\right) & (|p|>|q|)\\ \frac{2}{a\sqrt{q^2-p^2}}\coth^{-1}\left(\sqrt{\frac{q-p}{p+q}}\tan\frac{ax}{2}\right) & (|p|<|q|)\end{cases}$$
$$\int \frac{dx}{p^2+q^2\cos^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}}\tan^{-1}\left(\frac{p}{\sqrt{p^2+q^2}}\tan ax\right)$$
$$\int \frac{x}{1+\cos ax}\,dx = \frac{x}{a}\tan\frac{ax}{2} + \frac{2}{a^2}\ln\left|\cos\frac{ax}{2}\right|$$
$$\int \frac{x}{1-\cos ax}\,dx = -\frac{x}{a}\cot\frac{ax}{2} + \frac{2}{a^2}\ln\left|\sin\frac{ax}{2}\right|$$
$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad (|a|\neq|b|)$$
$$\int \tan ax\,dx = -\frac1a\ln|\cos ax| = \frac1a\ln|\sec ax|$$
$$\int \tan^2 ax\,dx = \frac1a\tan ax - x$$
$$\int \frac{dx}{p+q\tan ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)}\ln|q\sin ax+p\cos ax|$$
$$\int \sec ax\,dx = \frac1a\ln|\sec ax+\tan ax|$$
$$\int \sec^2 ax\,dx = \frac1a\tan ax$$
$$\int x\sec^2 ax\,dx = \frac{x}{a}\tan ax + \frac{1}{a^2}\ln|\cos ax|$$
$$\int \frac{dx}{1\pm\sec ax} = x\mp\frac1a\begin{cases}\tan\frac{ax}{2}\\ \cot\frac{ax}{2}\end{cases}$$
$$\int \csc ax\,dx = \frac1a\ln|\csc ax-\cot ax| = \frac1a\ln\left|\tan\frac{ax}{2}\right|$$
$$\int \csc^2 ax\,dx = -\frac1a\cot ax$$
$$\int x\csc^2 ax\,dx = -\frac{x}{a}\cot ax + \frac{1}{a^2}\ln|\sin ax|$$
$$\int \frac{dx}{1\pm\csc ax} = x - \frac{2}{a\cot\frac{ax}{2}\pm1}$$
$$\int \cot ax\,dx = \frac1a\ln|\sin ax|$$
$$\int \cot^2 ax\,dx = -\frac1a\cot ax - x$$
$$\int \frac{dx}{p+q\cot ax} = \frac{px}{p^2+q^2} - \frac{q}{a(p^2+q^2)}\ln|p\sin ax+q\cos ax|$$
$$\int \frac{dx}{\cos ax\pm\sin ax} = \frac{1}{a\sqrt2}\ln\left|\tan\left(\frac{ax}{2}\pm\frac{\pi}{8}\right)\right|$$
$$\int \frac{dx}{(\cos ax\pm\sin ax)^2} = \frac{1}{2a}\tan\left(ax\mp\frac{\pi}{4}\right)$$
$$\int \frac{\cot ax}{1\pm\cos ax}\,dx = \pm\frac{1}{2a}\ln\left|\tan\frac{ax}{2}\right| - \frac{1}{4a}\begin{cases}\tan^2\frac{ax}{2}\\ \cot^2\frac{ax}{2}\end{cases}$$
$$\int \frac{\sin ax}{\cos ax(1+\sin ax)}\,dx = \frac{1}{4a}\cot^2\left(\frac{ax}{2}+\frac{\pi}{4}\right) + \frac{1}{2a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|$$
$$\int \frac{\sin ax}{\cos ax(1-\sin ax)}\,dx = \frac{1}{4a}\tan^2\left(\frac{ax}{2}+\frac{\pi}{4}\right) - \frac{1}{2a}\ln\left|\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right|$$
$$\int \sin ax\cos ax\,dx = \frac{1}{2a}\sin^2 ax$$
$$\int \sin ax\cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} \qquad (|a|\neq|b|)$$
$$\int \frac{\sin ax}{\cos^n ax}\,dx = \frac{1}{a(n-1)\cos^{n-1}ax} \qquad (n\neq1)$$
$$\int \frac{\cos ax}{\sin^n ax}\,dx = -\frac{1}{a(n-1)\sin^{n-1}ax} \qquad (n\neq1)$$
$$\int \cos ax\cot ax\,dx = \frac1a\left(\cos ax + \ln\left|\tan\frac{ax}{2}\right|\right)$$
$$\int \sin ax\tan ax\,dx = \frac1a(\ln|\sec ax+\tan ax| - \sin ax)$$

Other

Error Function	$\mathrm{erf}(x) = \frac{2}{\sqrt\pi}\int_0^x e^{-t^2}\,dt$
Complementary Error Function	$\mathrm{erfc}(x) = 1-\mathrm{erf}(x) = \frac{2}{\sqrt\pi}\int_x^\infty e^{-t^2}\,dt$
Sine and Cosine Integrals	$\mathrm{Si}(x) = \int_0^x\frac{\sin t}{t}\,dt$; $\quad\mathrm{Ci}(x) = -\int_x^\infty\frac{\cos t}{t}\,dt$
Fresnel Integrals	$\mathrm{C}(x) = \int_0^x\cos\frac{\pi t^2}{2}\,dt$; $\quad\mathrm{S}(x) = \int_0^x\sin\frac{\pi t^2}{2}\,dt$
Exponential Integral	$\mathrm{Ei}(x) = \int_{-\infty}^x\frac{e^t}{t}\,dt$
Airy Functions	$\begin{cases}\mathrm{Ai}(x) = \frac1\pi\int_0^\infty\cos\left(\frac{t^3}{3}+xt\right)\,dt\\ \mathrm{Bi}(x) = \frac1\pi\int_0^\infty\left[\exp\left(-\frac{t^3}{3}+xt\right)+\sin\left(\frac{t^3}{3}+xt\right)\right]\,dt\end{cases}$
Gamma Function	$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$
Riemann Zeta function	$\zeta(x) = \frac{1}{\Gamma(x)}\int_0^\infty\frac{t^{x-1}}{e^t-1}\,dt$
Elliptic Integral, first kind	$\mathrm{F}(\phi,k) = \int_0^\phi\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$
Elliptic Integral, second kind	$\mathrm{E}(\phi,k) = \int_0^\phi\sqrt{1-k^2\sin^2\theta}\,d\theta$
Bessel Function, first kind	$J_n(x) = \frac1\pi\int_0^\pi\cos(nt-x\sin t)\,dt$
Modified Bessel Function, first kind	$\begin{cases}I_0(x) = \frac{1}{2\pi}\int_0^{2\pi}e^{x\cos\theta}\,d\theta\\ I_0(\sqrt{x^2+y^2}) = \frac{1}{2\pi}\int_0^{2\pi}e^{x\cos\theta+y\sin\theta}\,d\theta\end{cases}$
Delta Function	$\delta(t-s) = \frac{1}{2\pi}\int_{-\infty}^\infty e^{i\omega(t-s)}\,d\omega = \frac{1}{\pi^2}\int_{-\infty}^\infty\frac{dx}{(t-x)(s-x)}$

Inequalities: An integrable function f on $[a,b]$ is bounded on that interval. Thus there exist real numbers m and M such that $m\leq f(x)\leq M$ for all $x\in[a,b]$. Since the upper and lower sums of f over $[a,b]$ are therefore bounded by $m(b-a)$ and $M(b-a)$, respectively, we have that $m(b-a)\leq\int_a^b f(x)\,dx\leq M(b-a)$.
If $f(x)\leq g(x)$ for all $x\in[a,b]$, then $\int_a^b f(x)\,dx\leq\int_a^b g(x)\,dx$.
If $[c,d]$ is a subinterval of $[a,b]$ and $f(x)$ is non-negative for all $x\in[a,b]$, then $\int_c^d f(x)\,dx\leq\int_a^b f(x)\,dx$.
If f is Riemann-integrable on $[a,b]$, then the same is true for $|f|$, and $\left|\int_a^b f(x)\,dx\right|\leq\int_a^b|f(x)|\,dx$.
If f and g are Riemann-integrable, then so are f^2, g^2, and fg, and $\left(\int_a^b(fg)(x)\,dx\right)^2\leq\left(\int_a^b f^2(x)\,dx\right)\left(\int_a^b g^2(x)\,dx\right)$, known as the Cauchy-Schwartz inequality.

Inverse Trig. Functions

$$\int \sin^{-1}\frac{x}{a}\,dx = x\sin^{-1}\frac{x}{a} + \sqrt{a^2-x^2}$$
$$\int x\sin^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\sin^{-1}\frac{x}{a} + \frac{x}{4}\sqrt{a^2-x^2}$$
$$\int x^2\sin^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\sin^{-1}\frac{x}{a} + \frac19(x^2+2a^2)\sqrt{a^2-x^2}$$
$$\int \frac{1}{x^2}\sin^{-1}\frac{x}{a}\,dx = -\frac1x\sin^{-1}\frac{x}{a} - \frac1a\operatorname{sech}^{-1}\frac{x}{a}$$
$$\int \left(\sin^{-1}\frac{x}{a}\right)^2dx = x\left(\sin^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\sin^{-1}\frac{x}{a}$$
$$\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$$
$$\int x\cos^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2}$$
$$\int x^2\cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac19(x^2+2a^2)\sqrt{a^2-x^2}$$
$$\int \frac{1}{x^2}\cos^{-1}\frac{x}{a}\,dx = -\frac1x\cos^{-1}\frac{x}{a} + \frac1a\operatorname{sech}^{-1}\frac{x}{a}$$
$$\int \left(\cos^{-1}\frac{x}{a}\right)^2dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\cos^{-1}\frac{x}{a}$$
$$\int \tan^{-1}\frac{x}{a}\,dx = x\tan^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$$
$$\int x\tan^{-1}\frac{x}{a}\,dx = \frac12(a^2+x^2)\tan^{-1}\frac{x}{a} - \frac{ax}{2}$$
$$\int x^2\tan^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\tan^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$$
$$\int \frac{1}{x^2}\tan^{-1}\frac{x}{a}\,dx = -\frac1x\tan^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$$
$$\int \cot^{-1}\frac{x}{a}\,dx = x\cot^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$$
$$\int x\cot^{-1}\frac{x}{a}\,dx = \frac12(a^2+x^2)\cot^{-1}\frac{x}{a} + \frac{ax}{2}$$
$$\int x^2\cot^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cot^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$$
$$\int \frac{1}{x^2}\cot^{-1}\frac{x}{a}\,dx = -\frac1x\cot^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$$
$$\int \sec^{-1}\frac{x}{a}\,dx = x\sec^{-1}\frac{x}{a} - a\cosh^{-1}\left|\frac{x}{a}\right|$$
$$\int x\sec^{-1}\frac{x}{a}\,dx = \frac{x^2}{2}\sec^{-1}\frac{x}{a} - \frac{a}{2}\sqrt{x^2-a^2}$$
$$\int x^2\sec^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\sec^{-1}\frac{x}{a} - \frac{a}{6}x\sqrt{x^2-a^2} - \frac{a^3}{6}\cosh^{-1}\frac{x}{a}$$
$$\int \frac{1}{x^2}\sec^{-1}\frac{x}{a}\,dx = -\frac1x\sec^{-1}\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{ax}$$
$$\int \csc^{-1}\frac{x}{a}\,dx = x\csc^{-1}\frac{x}{a} + a\cosh^{-1}\frac{x}{a}$$
$$\int x\csc^{-1}\frac{x}{a}\,dx = \frac{x^2}{2}\csc^{-1}\frac{x}{a} + \frac{a}{2}\sqrt{x^2-a^2}$$
$$\int x^2\csc^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\csc^{-1}\frac{x}{a} + \frac{a}{6}x\sqrt{x^2-a^2} + \frac{a^3}{6}\cosh^{-1}\frac{x}{a}$$
$$\int \frac{1}{x^2}\csc^{-1}\frac{x}{a}\,dx = -\frac1x\csc^{-1}\frac{x}{a} - \frac{\sqrt{x^2-a^2}}{ax}$$

Exponential Functions

$$\int e^{ax}\,dx = \frac1a e^{ax}$$
$$\int a^{bx}\,dx = \frac{1}{b\ln a}a^{bx} \qquad (a>0,\ a\neq1)$$
$$\int xe^{ax}\,dx = \frac1a e^{ax}\left(x-\frac1a\right)$$
$$\int x^2e^{ax}\,dx = \frac1a e^{ax}\left(x^2-\frac{2x}{a}+\frac{2}{a^2}\right)$$
$$\int \frac{dx}{p+qe^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+qe^{ax})$$
$$\int \frac{dx}{(p+qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+qe^{ax})} - \frac{1}{ap^2}\ln(p+qe^{ax})$$
$$\int \frac{dx}{pe^{ax}+qe^{-ax}} = \begin{cases}\frac{1}{a\sqrt{pq}}\tan^{-1}\left(\sqrt{\frac{p}{q}}e^{ax}\right) & (pq>0)\\ -\frac{1}{a\sqrt{-pq}}\tanh^{-1}\left(\sqrt{\frac{-p}{q}}e^{ax}\right) & (pq<0)\end{cases}$$
$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$
$$\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}(a\sin bx-b\cos bx)}{a^2+b^2} - \frac{e^{ax}[(a^2-b^2)\sin bx-2ab\cos bx]}{(a^2+b^2)^2}$$
$$\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}(a\cos bx+b\sin bx)}{a^2+b^2} - \frac{e^{ax}[(a^2-b^2)\cos bx+2ab\sin bx]}{(a^2+b^2)^2}$$
$$\int xe^{ax^2}\,dx = \frac{1}{2a}e^{ax^2}$$

Logarithmic Functions

$$\int \ln ax\,dx = x\ln ax - x$$
$$\int \ln(ax+b)\,dx = \frac{(ax+b)\ln(ax+b)-(ax)}{a}$$
$$\int (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x$$
$$\int x\ln x\,dx = \frac{x^2}{2}\left(\ln x - \frac12\right)$$
$$\int x^m\ln x\,dx = x^{m+1}\left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2}\right) \qquad (\text{for } m\neq-1)$$
$$\int \frac{\ln x}{x}\,dx = \frac12(\ln x)^2$$
$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac1x$$
$$\int \frac{(\ln x)^n}{x}\,dx = \frac{(\ln x)^{n+1}}{n+1} \qquad (\text{for } n\neq-1)$$
$$\int \frac{\ln x}{x^m}\,dx = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2x^{m-1}} \qquad (\text{for } m\neq1)$$
$$\int \frac{dx}{x\ln x} = \ln|\ln x|$$
$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}} \qquad (\text{for } n\neq1)$$
$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\tan^{-1}\frac{x}{a}$$
$$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) - 2x + 2a\tanh^{-1}\frac{x}{a}$$
$$\int \frac{x}{x^2+a^2}\ln(x^2+a^2)\,dx = \frac14\left[\ln(x^2+a^2)\right]^2$$
$$\int \sin(\ln x)\,dx = \frac{x}{2}[\sin(\ln x) - \cos(\ln x)]$$
$$\int \cos(\ln x)\,dx = \frac{x}{2}[\sin(\ln x) + \cos(\ln x)]$$
$$\int e^x\left(x\ln x - x - \frac1x\right)dx = e^x(x\ln x - x - \ln x)$$
$$\int \frac{1}{e^x}\left(\frac1x - \ln x\right)dx = \frac{\ln x}{e^x}$$
$$\int e^x\left(\frac{1}{\ln x} - \frac{1}{x\ln^2 x}\right)dx = \frac{e^x}{\ln x}$$

Arc Length of Curve: $L = \int_a^b\sqrt{1+[f'(x)]^2}\,dx$.
Surface of Revolution: $A = 2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$.
Volume of Revolution: $V = \pi\int_a^b[f(x)]^2\,dx$.
Improper Integral: $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$, $\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$, or $\int_a^b f(x)\,dx = \lim_{d\to b^-}\int_a^d f(x)\,dx$, $\int_b^c f(x)\,dx = \lim_{d\to a^+}\int_d^c f(x)\,dx$, for f unbounded at $x=b$.
Principle Value: $⨍_a^c f(x)\,dx = \lim_{\varepsilon\to0^+}\left[\int_a^{b-\varepsilon}f(x)\,dx + \int_{b+\varepsilon}^c f(x)\,dx\right]$ for $\int_a^b f(x)\,dx = \pm\infty$, $a<b$ and $\int_b^c f(x)\,dx = \mp\infty$, $c>b$. $⨍_{-\infty}^\infty f(x)\,dx = \lim_{a\to\infty}\int_{-a}^a f(x)\,dx$ for $\int_{-\infty}^0 f(x)\,dx = \pm\infty$ and $\int_0^\infty f(x)\,dx = \mp\infty$.

Hyperbolic Trigonometric Functions

$$\int \sinh ax\,dx = \frac1a\cosh ax$$
$$\int x\sinh ax\,dx = \frac1a x\cosh ax - \frac{1}{a^2}\sinh ax$$
$$\int x^2\sinh ax\,dx = \left(\frac{x^2}{a}+\frac{2}{a^3}\right)\cosh ax - \frac{2x}{a^2}\sinh ax$$
$$\int \sinh^2 ax\,dx = \frac{1}{4a}\sinh 2ax - \frac{x}{2}$$
$$\int x\sinh^2 ax\,dx = \frac{x}{4a}\sinh 2ax - \frac{1}{8a^2}\cosh 2ax - \frac{x^2}{4}$$
$$\int \cosh ax\,dx = \frac1a\sinh ax$$
$$\int x\cosh ax\,dx = \frac1a x\sinh ax - \frac{1}{a^2}\cosh ax$$
$$\int x^2\cosh ax\,dx = \left(\frac{x^2}{a}+\frac{2}{a^3}\right)\sinh ax - \frac{2x}{a^2}\cosh ax$$
$$\int \cosh^2 ax\,dx = \frac{1}{4a}\sinh 2ax + \frac{x}{2}$$
$$\int x\cosh^2 ax\,dx = \frac{x}{4a}\sinh 2ax - \frac{1}{8a^2}\cosh 2ax + \frac{x^2}{4}$$
$$\int \tanh ax\,dx = \frac1a\ln|\cosh ax|$$
$$\int \tanh^2 ax\,dx = x - \frac1a\tanh ax$$
$$\int x\tanh^2 ax\,dx = \frac{1}{a^2}\ln\cosh ax - \frac{x}{a}\tanh ax - \frac{x^2}{2}$$
$$\int \coth ax\,dx = \frac1a\ln|\sinh ax|$$
$$\int \operatorname{csch} ax\,dx = \frac1a\ln\left|\tanh\frac{ax}{2}\right|$$
$$\int \operatorname{sech} ax\,dx = \frac2a\tan^{-1}e^{ax}$$
$$\int \sinh ax\sinh bx\,dx = \frac{a}{a^2-b^2}\sinh bx\cosh ax - \frac{b}{a^2-b^2}\cosh bx\sinh ax \qquad (|a|\neq|b|)$$
$$\int \cosh ax\cosh bx\,dx = \frac{a}{a^2-b^2}\sinh ax\cosh bx - \frac{b}{a^2-b^2}\sinh bx\cosh ax \qquad (|a|\neq|b|)$$
$$\int \cosh ax\sinh bx\,dx = \frac{a}{a^2-b^2}\sinh ax\sinh bx - \frac{b}{a^2-b^2}\cosh ax\cosh bx \qquad (|a|\neq|b|)$$
$$\int \sinh ax\sin bx\,dx = \frac{a}{a^2+b^2}\cosh ax\sin bx - \frac{b}{a^2+b^2}\sinh ax\cos bx$$
$$\int \sinh ax\cos bx\,dx = \frac{a}{a^2+b^2}\cosh ax\cos bx + \frac{b}{a^2+b^2}\sinh ax\sin bx$$
$$\int \cosh ax\sin bx\,dx = \frac{a}{a^2+b^2}\sinh ax\sin bx - \frac{b}{a^2+b^2}\cosh ax\cos bx$$
$$\int \cosh ax\cos bx\,dx = \frac{a}{a^2+b^2}\sinh ax\cos bx + \frac{b}{a^2+b^2}\cosh ax\sin bx$$

Inverse Hyperbolic Functions

$$\int \sinh^{-1}\frac{x}{a}\,dx = x\sinh^{-1}\frac{x}{a} - \sqrt{x^2+a^2}$$
$$\int x\sinh^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}+\frac{a^2}{4}\right)\sinh^{-1}\frac{x}{a} - \frac{x}{4}\sqrt{x^2+a^2}$$
$$\int x^2\sinh^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\sinh^{-1}\frac{x}{a} + \frac19(2a^2-x^2)\sqrt{x^2+a^2}$$
$$\int \frac{1}{x^2}\sinh^{-1}\frac{x}{a}\,dx = -\frac1x\sinh^{-1}\frac{x}{a} - \frac1a\operatorname{csch}^{-1}\frac{x}{a}$$
$$\int \cosh^{-1}\frac{x}{a}\,dx = x\cosh^{-1}\frac{x}{a} - \sqrt{x^2-a^2}$$
$$\int x\cosh^{-1}\frac{x}{a}\,dx = \frac14(2x^2-a^2)\cosh^{-1}\frac{x}{a} \mp \frac{x}{4}\sqrt{x^2-a^2}$$
$$\int x^2\cosh^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cosh^{-1}\frac{x}{a} \mp \frac19(x^2+2a^2)\sqrt{x^2-a^2}$$
$$\int \frac{1}{x^2}\cosh^{-1}\frac{x}{a}\,dx = -\frac1x\cosh^{-1}\frac{x}{a} \pm \frac1a\operatorname{csch}^{-1}\frac{x}{a}$$
$$\int \tanh^{-1}\frac{x}{a}\,dx = x\tanh^{-1}\frac{x}{a} + \frac{a}{2}\ln|a^2-x^2| \qquad (|x|<|a|)$$
$$\int x\tanh^{-1}\frac{x}{a}\,dx = \frac{a}{2}x + \frac12(x^2-a^2)\tanh^{-1}\frac{x}{a}$$
$$\int x^2\tanh^{-1}\frac{x}{a}\,dx = \frac{a}{6}x^2 + \frac{x^3}{3}\tanh^{-1}\frac{x}{a} + \frac{a^3}{6}\ln(a^2-x^2)$$
$$\int \frac{1}{x^2}\tanh^{-1}\frac{x}{a}\,dx = -\frac1x\tanh^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2}{a^2-x^2}\right)$$
$$\int \coth^{-1}\frac{x}{a}\,dx = x\coth^{-1}\frac{x}{a} + \frac{a}{2}\ln|x^2-a^2| \qquad (|x|>|a|)$$
$$\int x\coth^{-1}\frac{x}{a}\,dx = \frac{a}{2}x + \frac12(x^2-a^2)\coth^{-1}\frac{x}{a}$$
$$\int x^2\coth^{-1}\frac{x}{a}\,dx = \frac{a}{6}x^2 + \frac{x^3}{3}\coth^{-1}\frac{x}{a} + \frac{a^3}{6}\ln(x^2-a^2)$$
$$\int \frac{1}{x^2}\coth^{-1}\frac{x}{a}\,dx = -\frac1x\coth^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2}{x^2-a^2}\right)$$
$$\int \operatorname{sech}^{-1}\frac{x}{a}\,dx = x\operatorname{sech}^{-1}\frac{x}{a} - a\tan^{-1}\frac{x\sqrt{\frac{a-x}{a+x}}}{x-a} \qquad (0<x<a)$$
$$\int \operatorname{csch}^{-1}\frac{x}{a}\,dx = x\operatorname{csch}^{-1}\frac{x}{a} + a\sinh^{-1}\frac{x}{a} \qquad (0<x<a)$$

Definite Integrals

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2} \qquad (a>0)$$
$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2} \qquad (a>0)$$
$$\int_0^\infty \frac{e^{-ax}\sin bx}{x}\,dx = \tan^{-1}\frac{b}{a} \qquad (a>0)$$
$$\int_0^\infty xe^{-ax}\sin bx\,dx = \frac{2ab}{(a^2+b^2)^2} \qquad (a>0)$$
$$\int_0^\infty xe^{-ax}\cos bx\,dx = \frac{a^2-b^2}{(a^2+b^2)^2} \qquad (a>0)$$
$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a} \qquad (a,b>0)$$
$$\int_0^\infty e^{-ax^2}\,dx = \frac12\sqrt{\frac{\pi}{a}} \qquad (a>0)$$
$$\int_{-\infty}^\infty e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \qquad (a>0)$$
$$\int_{-\infty}^\infty e^{-ax^2}e^{-2bx}\,dx = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{a}} \qquad (a>0)$$
$$\int_{-\infty}^\infty xe^{-a(x-b)^2}\,dx = b\sqrt{\frac{\pi}{a}}$$
$$\int_{-\infty}^\infty x^2e^{-ax^2}\,dx = \frac12\sqrt{\frac{\pi}{a^3}} \qquad (a>0)$$
$$\int_0^\infty x^ne^{-ax^2}\,dx = \begin{cases}\frac12\Gamma\left(\frac{n+1}{2}\right)/a^{\frac{n+1}{2}} & (n>-1,\ a>0)\\ \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} & (n=2k,\ k\in\mathbb{Z},\ a>0)\\ \frac{k!}{2a^{k+1}} & (n=2k+1,\ k\in\mathbb{Z},\ a>0)\end{cases}$$
$$\int_0^\infty e^{-ax^p}\,dx = a^{-1/p}\Gamma(1+1/p) \qquad (a,p>0)$$
$$\int_0^\infty e^{-ax}\,dx = \frac1a$$
$$\int_0^\infty x^ne^{-ax}\,dx = \begin{cases}\frac{\Gamma(n+1)}{a^{n+1}} & (n>-1,\ a>0)\\ \frac{n!}{a^{n+1}} & (n\in\mathbb{N},\ a>0)\end{cases}$$
$$\int_0^\infty \frac{x}{e^x-1}\,dx = \frac{\pi^2}{6}$$
$$\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{\pi^4}{15}$$
$$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n)\zeta(n)$$
$$\int_0^\infty \frac{x}{e^x+1}\,dx = \frac{\pi^2}{12}$$
$$\int_0^\infty \frac{\sin ax}{e^{2\pi x}-1}\,dx = \frac14\coth\frac{a}{2} - \frac{1}{2a}$$
$$\int_0^\infty \left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x} = \gamma \approx 0.577\,216$$
$$\int_0^\infty \frac{e^{-x^2}-e^{-x}}{x}\,dx = \frac{\gamma}{2}$$
$$\int_0^\infty \left(\frac{1}{e^x-1}-\frac{e^{-x}}{x}\right)dx = \gamma$$
$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x\sec px}\,dx = \frac12\ln\left(\frac{b^2+p^2}{a^2+p^2}\right)$$
$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x\csc px}\,dx = \tan^{-1}\frac{b}{p} - \tan^{-1}\frac{a}{p}$$
$$\int_0^\infty \frac{e^{-ax}(1-\cos x)}{x^2}\,dx = \cot^{-1}a - \frac{a}{2}\ln\left(a^2+1\right)$$

Definite Integrals, Cont.

$$\int_0^\pi \sin mx\sin nx\,dx = \begin{cases}0 & m,n\in\mathbb{Z},\ m\neq n\\ \frac{\pi}{2} & m,n\in\mathbb{Z},\ m=n\end{cases}$$
$$\int_0^\pi \cos mx\cos nx\,dx = \begin{cases}0 & m,n\in\mathbb{Z},\ m\neq n\\ \frac{\pi}{2} & m,n\in\mathbb{Z},\ m=n\end{cases}$$
$$\int_0^\pi \sin mx\cos nx\,dx = \begin{cases}0 & m,n\in\mathbb{Z},\ m+n\ \text{odd}\\ \frac{2m}{m^2-n^2} & m,n\in\mathbb{Z},\ m+n\ \text{even}\end{cases}$$
$$\int_0^{\pi/2}\sin^2 x\,dx = \int_0^{\pi/2}\cos^2 x\,dx = \frac{\pi}{4}$$
$$\int_0^{\pi/2}\sin^n x\,dx = \int_0^{\pi/2}\cos^n x\,dx = \begin{cases}\frac{(n-1)!!}{n!!}\frac{\pi}{2} & n\ \text{even},\ n\geq2\\ \frac{(n-1)!!}{n!!} & n\ \text{odd},\ n\geq3\end{cases}$$
$$\int_0^\pi \cos^{2n}x\,dx = \pi\frac{(2n)!}{2^{2n}(n!)^2}$$
$$\int_0^{\pi/2}\sin^{2p-1}x\cos^{2q-1}x\,dx = \frac{\Gamma(p)\Gamma(q)}{2\Gamma(p+q)}$$
$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$
$$\int_0^\infty \frac{\sin^2 px}{x^2}\,dx = \frac{\pi p}{2}$$
$$\int_0^\infty \frac{\sin px\cos qx}{x}\,dx = \begin{cases}0 & p>q>0\\ \pi/2 & 0<p<q\\ \pi/4 & p=q>0\end{cases}$$
$$\int_0^\infty \frac{\sin px\cos qx}{x^2}\,dx = \begin{cases}\pi p/2 & 0<p\leq q\\ \pi q/2 & p\geq q>0\end{cases}$$
$$\int_0^\infty \frac{1-\cos px}{x^2}\,dx = \frac{\pi p}{2}$$
$$\int_0^\infty \frac{\cos px-\cos qx}{x}\,dx = \ln\frac{q}{p}$$
$$\int_0^\infty \frac{\cos px-\cos qx}{x^2}\,dx = \frac{\pi(q-p)}{2}$$
$$\int_0^\infty \frac{\cos mx}{x^2+a^2}\,dx = \frac{\pi}{2a}e^{-ma}$$
$$\int_0^\infty \frac{x\sin mx}{x^2+a^2}\,dx = \frac{\pi}{2}e^{-ma}$$
$$\int_0^\infty \frac{\sin mx}{x(x^2+a^2)}\,dx = \frac{\pi}{2a^2}(1-e^{-ma})$$
$$\int_0^{2\pi}\frac{dx}{a+b\sin x} = \int_0^{2\pi}\frac{dx}{a+b\cos x} = \frac{2\pi}{\sqrt{a^2-b^2}}$$
$$\int_0^{\pi/2}\frac{dx}{a+b\cos x} = \frac{\cos^{-1}b/a}{\sqrt{a^2-b^2}}$$
$$\int_0^{2\pi}\frac{dx}{(a+b\sin x)^2} = \int_0^{2\pi}\frac{dx}{(a+b\cos x)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}}$$
$$\int_0^{2\pi}\frac{dx}{1-2a\cos x+a^2} = \frac{2\pi}{1-a^2} \qquad (0<a<1)$$
$$\int_0^\pi \frac{x\sin x}{1-2a\cos x+a^2}\,dx = \begin{cases}\frac{\pi}{a}\ln(1+a) & |a|<1\\ \pi\ln(1+1/a) & |a|>1\end{cases}$$
$$\int_0^\pi \frac{\cos mx}{1-2a\cos x+a^2}\,dx = \frac{\pi a^m}{1-a^2} \qquad (a^2<1,\ m\in\mathbb{Z})$$
$$\int_0^\infty \cos(ax^2)\,dx = \int_0^\infty\sin(ax^2)\,dx = \frac12\sqrt{\frac{\pi}{2a}} \qquad (a>0)$$
$$\int_0^\infty \sin(ax^n)\,dx = \frac{1}{na^{1/n}}\Gamma(1/n)\sin\frac{\pi}{2n} \qquad (n>1)$$
$$\int_0^\infty \cos(ax^n)\,dx = \frac{1}{na^{1/n}}\Gamma(1/n)\cos\frac{\pi}{2n} \qquad (n>1)$$
$$\int_0^\infty \frac{\sin x}{\sqrt x}\,dx = \int_0^\infty\frac{\cos x}{\sqrt x}\,dx = \sqrt{\frac{\pi}{2}}$$
$$\int_0^\infty \frac{\sin x}{x^p}\,dx = \frac{\pi}{2\Gamma(p)\sin(p\pi/2)} \qquad (0<p<1)$$
$$\int_0^\infty \frac{\cos x}{x^p}\,dx = \frac{\pi}{2\Gamma(p)\cos(p\pi/2)} \qquad (0<p<1)$$
$$\int_0^\infty \frac{\sin^3 x}{x^3}\,dx = \frac{3\pi}{8}$$
$$\int_0^\infty \frac{\sin^4 x}{x^4}\,dx = \frac{\pi}{3}$$
$$\int_0^\infty \frac{\tan x}{x}\,dx = \frac{\pi}{2}$$
$$\int_0^{\pi/2}\frac{dx}{1+\tan^m x} = \frac{\pi}{4}$$
$$\int_0^1 \frac{\sin^{-1}x}{x}\,dx = \frac{\pi}{2}\ln2$$
$$\int_0^\infty \left(\frac{1}{1+x^2}-\cos x\right)\frac{dx}{x} = \gamma$$
$$\int_0^\infty \frac{\tan^{-1}px-\tan^{-1}qx}{x}\,dx = \frac{\pi}{2}\ln\frac{p}{q}$$
$$\int_0^1 a^xb^{1-x}\,dx = \frac{a-b}{\ln a-\ln b} \qquad (a,b>0,\ a\neq b)$$
$$\int_0^1 x^m(\ln x)^n\,dx = \frac{(-1)^nn!}{(m+1)^{n+1}} \qquad (m>-1,\ n\in\mathbb{N})$$
$$\int_0^1 \left(\ln\frac1x\right)^p\,dx = p!$$
$$\int_0^1 \frac{\ln x}{1+x}\,dx = -\frac{\pi^2}{12}$$
$$\int_0^1 \frac{\ln x}{1-x}\,dx = -\frac{\pi^2}{6}$$
$$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$
$$\int_0^1 \frac{\ln(1-x)}{x}\,dx = -\frac{\pi^2}{6}$$
$$\int_0^1 \ln x\ln(1+x)\,dx = 2-2\ln2-\frac{\pi^2}{12}$$
$$\int_0^1 \ln x\ln(1-x)\,dx = 2-\frac{\pi^2}{6}$$
$$\int_0^1 \frac{\ln x}{1-x^2}\,dx = -\frac{\pi^2}{8}$$
$$\int_0^\infty \frac{x^{p-1}\ln x}{1+x}\,dx = -\pi^2\csc p\pi\cot p\pi \qquad (0<p<1)$$
$$\int_0^1 \frac{x^m-x^n}{\ln x}\,dx = \ln\frac{m+1}{n+1} \qquad (n\neq-1)$$
$$\int_0^\infty e^{-x}\ln x\,dx = -\gamma$$
$$\int_0^\infty e^{-x^2}\ln x\,dx = -\frac{\sqrt\pi}{4}(\gamma+2\ln2)$$
$$\int_0^\infty \ln\left(\frac{e^x+1}{e^x-1}\right)dx = \frac{\pi^2}{4}$$
$$\int_0^{\pi/2}\ln\sin x\,dx = \int_0^{\pi/2}\ln\cos x\,dx = -\frac{\pi}{2}\ln2$$
$$\int_0^{\pi/2}(\ln\sin x)^2\,dx = \int_0^{\pi/2}(\ln\cos x)^2\,dx = \frac{\pi}{2}(\ln2)^2 + \frac{\pi^3}{24}$$
$$\int_0^\pi x\ln\sin x\,dx = -\frac{\pi^2}{2}\ln2$$
$$\int_0^{\pi/2}\sin x\ln\sin x\,dx = \ln2-1$$
$$\int_0^{2\pi}\ln(a+b\sin x)\,dx = \int_0^{2\pi}\ln(a+b\cos x)\,dx = 2\pi\ln(a+\sqrt{a^2-b^2})$$
$$\int_0^\pi \ln(a+b\cos x)\,dx = \pi\ln\left(\frac{a+\sqrt{a^2-b^2}}{2}\right)$$
$$\int_0^\pi \ln(a^2-2ab\cos x+b^2)\,dx = \begin{cases}2\pi\ln a & a\geq b>0\\ 2\pi\ln b & b\geq a>0\end{cases}$$
$$\int_0^{\pi/4}\ln(1+\tan x)\,dx = \frac{\pi}{8}\ln2$$
$$\int_0^{\pi/2}\sec x\ln\left(\frac{1+b\cos x}{1+a\cos x}\right)dx = \frac12\left[(\cos^{-1}a)^2-(\cos^{-1}b)^2\right]$$
$$\int_0^\infty \frac{\sin ax}{\sinh bx}\,dx = \frac{\pi}{2b}\tanh\frac{a\pi}{2b}$$
$$\int_0^\infty \frac{\cos ax}{\cosh bx}\,dx = \frac{\pi}{2b}\operatorname{sech}\frac{a\pi}{2b}$$
$$\int_0^\infty \frac{x}{\sinh ax}\,dx = \frac{\pi^2}{4a^2}$$
$$\int_0^\infty \frac{x^n}{\sinh ax}\,dx = \frac{2^{n+1}-1}{2^na^{n+1}}\Gamma(n+1)\zeta(n+1)$$
$$\int_0^\infty \frac{\sinh ax}{e^{bx}+1}\,dx = \frac{\pi}{2b}\csc\frac{a\pi}{b} - \frac{1}{2a}$$
$$\int_0^\infty \frac{\sinh ax}{e^{bx}-1}\,dx = \frac{1}{2a} - \frac{\pi}{2b}\cot\frac{a\pi}{b}$$
$$\int_0^\infty \frac{dx}{x^2+a^2} = \frac{\pi}{2a}$$
$$\int_0^\infty \frac{dx}{(1+x)x^a} = \frac{\pi}{\sin a\pi} \qquad (0<a<1)$$
$$\int_0^\infty \frac{dx}{(1-x)x^a} = -\frac{\pi}{\tan a\pi} \qquad (0<a<1)$$
$$\int_0^a \frac{dx}{\sqrt{a^2-x^2}} = \frac{\pi}{2}$$
$$\int_0^a \sqrt{a^2-x^2}\,dx = \frac{\pi a^2}{4}$$
$$\int_0^1 x^p(1-x)^q\,dx = \frac{p!q!}{(p+q+1)!} \qquad (p,q\in\mathbb{N})$$